Presentation of the UNESWA CGE Model

By: Professor Chokri Thabet
Overview

1. The Social Accounting Matrix: the SAM
2. Theoretical Structure of the core CGE Model
3. The definition of the Business As Usual Scenario
The Social Accounting Matrix
What is a SAM?

A SAM is a comprehensive, economy-wide data framework.

A square matrix with identical row and column accounts where each cell shows payment (at current prices) from its column account to its row account.

Used for descriptive purposes and as the key data input for CGE models.

“Social” → often focus on incomes and spending of households of different types. Disaggregation and classification of accounts vary widely across different SAMs.

Column totals = row totals.

First SAM developed for the UK in 1962 by Richard Stone, Nobel laureate for development of national income accounting.
The SAM distinguishes between current (flow) transactions and capital account transactions, where the latter alter the stocks of assets and liabilities of agents.

Firms are owned by households, or government, or the rest of the world. They use factors and intermediate inputs to produce goods and services for the market.
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Firms are owned by households, or government, or the rest of the world. They use factors and intermediate inputs to produce goods and services for the market.
The SAM is a representation of this basic idea.

Two functions:

- A description of an economy
- A basis for modelling

The core of a CGE is the Social Accounting Matrix or SAM.

This is a single-entry accounting representation of the flow of goods and services and payment between sectors, classes of economic actor and other accounts.
The Institutions of a SAM

Households (distinguished by type)

- Households supply factors of production (capital, land and labour) to firms; consume goods and services in the market; pay taxes to and receive subsidies from government; consume public goods; make net current transfers to the rest of the world; save and invest.
Institutions (contd.)

Government
- Levies taxes on households, firms, and commodities. Undertakes current consumption; makes transfers to households, firms, and the rest of the world; saves and invests (in sector specific capital and public infrastructure).

The rest of the world
- Supplies goods to domestic markets (imports) and consumes domestic output (exports); makes net transfers (remittances, grants, net interest payments); provides saving.
Basic Balance Requirements

Commodity Balance
  For each commodity and factor: \( \text{supply} = \text{demand} \)

Flow of Funds Balance
  For each institution: \( \text{total income} = \text{total expenditure} \)

Macroeconomic Balance
  Balance of Payments
  Saving = Investment
## Stylized SAM

### Table. Stylized SAM

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<th>Explanation</th>
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<th>k</th>
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### Table. Notation in SAM

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<td>r</td>
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# SAM Structure

<table>
<thead>
<tr>
<th>Activities</th>
<th>Commod</th>
<th>Factors</th>
<th>Institutions</th>
<th>Capital account</th>
<th>Rest of World</th>
<th>Total</th>
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<tr>
<td>Activities</td>
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<td>Hh and gov cons</td>
<td>Investment</td>
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<td>GNP at factor cost</td>
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<td>Institution</td>
<td>Indirect taxes</td>
<td>Tariff and indirect taxes</td>
<td>Lab income, Dist profit and undist profit</td>
<td>Transfers</td>
<td>Transfers</td>
<td>Institution income</td>
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<tr>
<td>Capital Account</td>
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<td></td>
<td>Hh, firm and gov savings</td>
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<td>Capital transfers</td>
<td>Total savings</td>
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<tr>
<td>Rest of world</td>
<td>Imports</td>
<td>Factor payments</td>
<td>Current transfers abroad</td>
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<td>Imports</td>
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<tr>
<td>Total</td>
<td>Production</td>
<td>Domestic supply</td>
<td>Factor outlay</td>
<td>Institution expenditure</td>
<td>Tot investment</td>
<td>Foreign exchange earnings</td>
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</table>
Theoretical Structure of the Core Model
1. Prices Definition

Import Price: 1.

The import price in LCU (local-currency units) is the price paid by domestic users for imported commodities (exclusive of the sales tax). It is a transformation of the world price of these imports, considering the exchange rate and import tariffs plus transaction costs (the cost of trade inputs needed to move the commodity from the border to the demander) per unit of the import.

\[ PIMP_{C,R,t} = pwm_{C,R,t} \cdot (1 + tm_{C,R,t}) \cdot EXR_t + \sum_{CT} (PDEM_{CT,t} \cdot icm_{C,i}) \]
The export price: in LCU is the price received by domestic producers when they sell their output in export markets. The tax and the cost of trade inputs reduce the price received by the domestic producers of exports. The domain of the equation is the set of exported commodities, all of which are produced domestically.

\[
P_{\text{EXP}}_{C,R,t} = pwe_{C,R,t} \cdot (1 - te_{C,R,t}) \cdot EXR_t - \sum_{CT} (PDEM_{CT,t} \cdot iC_{CT,C})
\]
Prices Definition

**Demand Price of Domestic Non traded Goods:** The model includes distinct prices for domestic output that is used domestically. In the presence of transaction costs, it distinguishes between prices paid by demanders and those received by suppliers.

\[ PD_{C,t} = PDS_{C,t} + \sum_{CT}(PDEM_{CT,t}.cd_{CT,C}) \]

**Absorption:** defined as total domestic spending on a commodity at domestic demander prices. Absorption is expressed as the sum of spending on domestic output and imports at the demand prices, \( PD \) and \( PIMP \). The prices \( PD \) and \( PIMP \) include the cost of trade inputs but exclude the commodity sales tax.

\[ PDEM_{C,t}(1 - TQ_{C,t}) DEM_{C,t} = PD_{C,t}.D_{C,t} + \sum_{R}(PIMP_{C,R,t}.IMP_{C,R,t}) \]
Marketed Output Value: For each domestically produced commodity, the marketed output value at producer prices is stated as the sum of the values of domestic sales and exports. Domestic sales and exports are valued at the prices received by the suppliers, PDS and PEXP, both of which have been adjusted downwards to account for the cost of trade inputs.

\[ PYC_{c,t} \cdot YC_{c,t} = PDS_{c,t} \cdot D_{c,t} + \sum_R (PEXP_{c,r,t} \cdot EXP_{c,r,t}) \]

Output Price: The gross revenue per activity unit, the activity price, is the return from selling the output or outputs of the activity, defined as yields per activity unit multiplied by activity-specific commodity prices, summed over all commodities. This allows for the fact that activities may produce multiple commodities.

\[ PYA_{A,t} = \sum_C (PYAC_{A,c,t} \cdot \theta_{A,c}) \]
**Prices Definition**

**Price of aggregate intermediate input:** The activity-specific aggregate intermediate input price shows the cost of disaggregated intermediate inputs per unit of aggregate intermediate input. It depends on composite commodity prices and intermediate input coefficients, which show the quantity of input commodity $c$ per unit of aggregate intermediate input.

$$PINTA_{A,t} = \sum_{C} (PDEM_{C,t} \cdot ic_{A,C})$$

**Value-added Price:** For each activity, total revenue net of taxes is fully exhausted by payments for value-added and intermediate inputs.

$$PVA_{A,t} \cdot VA_{A,t} = PYA_{A,t} \cdot (1 - tA_{A,t}) \cdot YA_{A,t} - PINTA_{A,t} \cdot INT_{A,t}$$
Prices Definition

**Consumer Price Index:**

\[
CPI_t = \sum_C cwt_s_C \cdot PDEM_{C,t}
\]

**Domestic Producer Price Index:**

\[
DPI_t = \sum_{CD} dwts_{CD} \cdot PDS_{CD,t}
\]

**GDP Definition:** The Gross Domestic Product is the sum of the gross value added by all resident producers in the economy

\[
GDP_t = \sum_A VA_{A,t}
\]
2. Production Block

Production is carried out by activities that are assumed to maximize profits subject to their technology, taking prices (for their outputs, intermediate inputs, and factors) as given.

It acts in a perfectly competitive setting.

The CGE model includes the first-order conditions for profit-maximization by producers. Producers choose the optimal bundle between values added and aggregated intermediate inputs, which is modeled by the Leontief function.
The production and trade block covers four categories:

(i) domestic production and input use;

(ii) the allocation of domestic output to home consumption, the domestic market, and exports;

(iii) the aggregation of supply to the domestic market (from imports and domestic output sold domestically); and

(iv) the definition of the demand for trade inputs that is generated by the distribution process.
**Demand for Aggregated Intermediate Input**

\[ INT_{A,t} = inta_A \cdot YA_{A,t} \]

**Demand for Aggregate Value-Added**

\[ VA_{A,t} = iva_A \cdot YA_{A,t} \]
2.1. Value-added and Factor Demands

Capital Demand

\[ K_{A,t}^{AGG} = VA_{A,t} \cdot A_{A,t}^{\nu A} (\sigma_{A}^{\nu A} - 1) \cdot (a^{\nu A} \cdot \frac{PVA_{A,t}}{PK_{A,t}}) \sigma_{A}^{\nu A} \]

Aggregated Labor Demand:

\[ L_{A,t}^{AGG} = VA_{A,t} \cdot A_{A,t}^{\nu A} (\sigma_{A}^{\nu A} - 1) \cdot (b^{\nu A} \cdot \frac{PVA_{A,t}}{W_{A,t}}) \sigma_{A}^{\nu A} \]

Skilled Labor Demand

\[ L_{SKL}(A,t) = L_{AGG}(A,t) \cdot \left( b_{L_{AGG}(A)} \cdot \frac{W_{AGG}(A,t)}{W_{SKL}(A,t)} \right) \sigma_{AGG}(A) \]

Unskilled labor demand

\[ L_{UNS}(A,t) = L_{AGG}(A,t) \cdot \left( b_{L_{UNSG}(A)} \cdot \frac{W_{AGG}(A,t)}{W_{UNS}(A,t)} \right) \sigma_{AGG}(A) \]
2.2 Factor market equilibrium

Value Added Equilibrium

\[ PV_{A,t} \cdot VA_{A,t} = PK_{A,t}^{AGG} \cdot K_{A,t}^{AGG} + W_{A,t}^{AGG} \cdot L_{A,t}^{AGG} \]

Labor market equilibrium

\[ W_{A,t}^{AGG} = W_t^{Bar} \]

Capital Market equilibrium

\[ K_{capital,A,t}^{AGG} = K_{A,t}^{AGG} \left( b_{capital,A}^{AGG} \cdot \frac{PK_{A,t}^{AGG}}{rK_{capital,A,t}} \right)^{\sigma_{K_A}^{AGG}} \]

\[ PK_{A,t}^{AGG} \cdot K_{A,t}^{AGG} = \sum_{capital} (rK_{capital,A,t} \cdot K_{capital,A,t}) \]
2.3 Exports vs. Domestic Supply

Output Transformation (CET) Function

The allocation of marketed domestic output to two alternative destinations: domestic sales and exports. With imperfect transformability between these two destinations.

\[
YC_{C,t} = A_C^{ac} \cdot \sum_A (b_{A,C}^{ac} \cdot YAC_{A,C,t}^{ac})^{1-a_C^{ac}}
\]

\[
YC_{C,t} = A_C^t \cdot (\sum_R (b_{C,R}^t \cdot EXP_{C,R,t}^{\sigma_C^t}) + (1 - \sum_R (b_{C,R}^t \cdot D_{C,t}^{\sigma_C^t}))^{\frac{1}{\sigma_C^t}}
\]
Exports vs. Domestic Supply

Output Transformation for Domestically Sold Outputs and Exports

\[ YC_{C,t} = D_{C,t} + \sum_{R} EXP_{C,R,t} \]

Export-Domestic Supply Ratio

Defines the optimal mix between exports and domestic sales

\[ EXP_{C,R,t} = D_{C,t} \cdot \left( \frac{PEXP_{C,R,t}}{PDS_{C,t}} \cdot \frac{1 - \sum_{RP} b_{C,R,t}^{t}}{b_{C,R,t}^{t}} \right)^{1/(\sigma_{t}^{t}-1)} \]
2. Demand

**Intermediate consumption** For each activity:

\[ IC_{C,A,t} = i c a_{C,A} \cdot INT_{A,t} \]

**Final consumption** It is assumed that each household maximizes its utility function subject to a consumption expenditure constraint.

\[
PDEM_{C,t} CH_{C,H,t} = PDEM_{C,t} \gamma_{C,H}^m + \beta_{C,H}^m (EXPH_{H,t} - \sum_{CP} PDEM_{CP,t} \gamma_{CP,H}^m - \sum_{A,CP} PYAC_{A,CP,t} \cdot \gamma_{A,CP,H}^h)
\]

\[
PYAC_{A,C,t} \cdot CHA_{A,C,H,t} = PYAC_{A,C,t} \cdot \gamma_{A,C,H}^h + \beta_{A,C,H}^h (EXPH_{H,t} - \sum_{CP} PDEM_{CP,t} \cdot \gamma_{CP,H}^m - \sum_{AP,CP} PYAC_{AP,CP,t} \cdot \gamma_{AP,CP,H}^h)
\]

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Government Consumption is supposed to be a fixed share

\[
PDEM_{C,t} \frac{G_{C,GOVF,t}}{EXPG_t - \sum_{INSDNSG} trnsfr_{INSDNSG,GOV,t} \cdot CPI_t}
\]

\[
= \frac{PDEM0_{C}. G0_{C,GOVF}}{(EXPG0 - \sum_{INSDNSG} trnsfr0_{INSDNSG,GOV}).}
\]

Capital Good Demand

\[
KG(C, t) = INV\text{TOTS}(t)a_{INV(C)} \left( \frac{PDEM(C, t)}{PINVTOT(t)} \right)^{\sigma_{\text{INV}(c)}}
\]
3. Local vs. Imported Demand

Imperfect substitutability between imports and domestic output sold domestically is captured by a CES aggregation function in which the composite commodity that is supplied domestically is produced by domestic and imported commodities entering this function as inputs.

\[
DEM_{c,t} = A_c^{DEM} \left( \sum_R b_{c,R}^{DEM} IMP_{c,R,t}^{DEM} \sigma_c^{DEM} \right) + (1 - \sum_R b_{c,R}^{DEM} D_{c,t}^{DEM})^{-1/\sigma_c^{DEM}}
\]
Import-Domestic Demand Ratio the optimal mix between imports and domestic output

\[
\frac{IMP_{C,R,t}}{D_{C,t}} = \left( \frac{PD_{C,t}}{PIMP_{C,R,t}} \cdot \frac{b_{C,R}^{DEM}}{1 - \sum_{R} b_{C,R}^{DEM}} \right) \frac{1}{1 + \sigma_{C}^{DEM}}
\]

Domestic demand

\[
DEM_{C,t} = D_{C,t} + \sum_{R} IMP_{C,R,t}
\]
Demand For Transactions Services

Total demand for trade inputs is the sum of the demands for these inputs that are generated by imports, exports, and domestic market sales.

\[
TR_{C,t} = \sum_{CP} icd_{C,CP} \cdot D_{CP,t} + \sum_{CP,R} icm_{C,CP} \cdot IMP_{CP,R,t} + \sum_{CP,R} ice_{C,CP} \cdot EXP_{CP,R,t}
\]
Factor Income

\[ YF(F, t) = \sum_A W(F, A, t)Q(F, A, t) \]

Factor incomes to domestic institutions  The income of each factor is split among domestic institutions in fixed shares after payment of direct factor taxes and transfers to the rest of the world.

\[ YIF_{INSF,F,t} = shif_{INSF,F} \cdot \left[ (1 - tf_{f,t}) \cdot YF_{F,t} - transf_{ROW,F,t} \cdot EXR_t \right] \]
Total incomes of domestic nongovernment institutions is the sum of factor incomes, transfers from other domestic nongovernment institutions, transfers from the government (indexed to the CPI), and transfers from the rest of the world.

\[
Y_{I_{\text{INSNG},t}} = \sum_f YIF_{\text{INSNG,F},t} + \sum_{\text{INSNGP}} TRII_{\text{INSNG,INSNGP},t} + \text{transfr}_{\text{INSNG,GOV},t} \cdot CPI_t + \text{transfr}_{\text{insdn,ROW},t} \cdot EXR_t
\]
**Institution Block**

**Transfers to Institutions from Institutions**

Transfers between domestic nongovernment institutions are paid as fixed shares of the total institutional incomes net of direct taxes and savings.

\[
TRII_{INSNG,INSNGP,t} = shii_{INSNG,INSNGP} \cdot \left(1 - t_{sav_{INSNGP,t}}\right) \cdot \left(1 - Tax_{Dir_{INSNGP,t}}\right) \cdot YI_{INSNGP,t}
\]
**Composite Commodity Market Equilibrium:** (Goods and Services market clearance)

This equation imposes equality between quantities supplied and demanded of the composite commodity. The composite commodity supply, $DEM$, drives demands for domestic marketed output, $QD$, and imports, $QM$. The market-clearing variables are the quantities of import supply, for the import side, and the two interrelated domestic prices, $PDD$ and $PDS$, for domestic market output.

$$DEM_{C,t} = \sum_A IC_{C,A,t} + \sum_H CH_{C,H,t} + \sum_{GOVF} G_{C,GOVF,t} + KG_{C,t} + qds t_{C,t} + TR_{C,t}$$
Current Account Balance for the Rest of the World: The current-account balance imposes equality between the country’s spending and its earning of foreign exchange. For the basic model version, foreign savings is fixed; the (real) exchange rate ($EXR$) serves the role of equilibrating variable to the current-account balance. The fact that all items except imports and exports are fixed means that, in effect, the trade deficit also is fixed. Alternatively, the exchange rate may be fixed and foreign savings unfixed. In this case, the trade deficit is free to vary.

\[
\sum_{CM,R} pwm_{CM,R,t} \cdot IMP_{CM,R,t} + \sum_{F} trnsfr_{ROW,F,t} + \sum_{INSD} trnsfr_{ROW,INSD,t} + INTF_{t}
\]

\[
= \sum_{CE,R} pwe_{CE,R,t} \cdot EXP_{CE,R,t} + \sum_{INSD} trnsfr_{INSD,ROW,t} + FSAV_{t}
\]
Government Balance The government balance imposes equality between current government revenue and the sum of current government expenditures (including interest payment) and savings

\[ GSAV_t = YG_t - EXPG_t - INV PUB_{tot_t} - INT_t \]
Savings-Investment Balance This equation states that total savings and total investment have to be equal. Total savings is the sum of savings from domestic nongovernment institutions, the government, and the rest of the world, with the last item converted into domestic currency. Total investment is the sum of the values of fixed investment (gross fixed capital formation) and stock changes. In the basic model version, the flexible variable, to-sav, performs the task of clearing this balance

\[ \text{FSAV}_t \cdot \text{EXR}_t = \sum_{\text{FCAP}, A} \text{PINVTOT}_t \cdot \text{INVP}_{\text{FCAP}, A, t} + \sum_{C} \text{PDEM}_{C, t} \cdot qdst_{C, t} - \sum_{\text{INSDNG}} \text{to_sav}_{\text{INSDNG}, t} \cdot (1 - \text{Tax}_{\text{Dir}, \text{INSDNG}, t}) \cdot \text{YI}_{\text{INSDNG}, t} + \text{GSAV}_t + \text{WALRAS}_t \]
Savings-Investment Balance

\[ \sum_{INSDNSG}^{to_sav_{INSDNSG,t}} \left(1 - Tax_{Dir_{INSDNSG,t}}\right) \cdot YI_{INSDNSG,t} + GSAV_t + WALRAS_t + FSAV_t \cdot EXR_t = \sum_{FCAP,A}^{PINVTOT_t \cdot INV_{FCAP,A,t}} + \sum_{C}^{PDEM_{C,t} \cdot qdst_{C,t}} \]
Dynamics

**Capital Accumulation**

\[ K(A, t) = (1 - \delta)K(FCAP, A, t - 1) + INV(A, t) \]

**Labor supply**

\[ LS(SKL, t) = LSI(SKL, t - 1)(1 + g_{L(t)}) \]